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VORTEX–EXCITED VIBRATIONS IN BUNDLED CONDUCTORS: A MATHEMATICAL MODEL

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Wind-excited vibrations generated by vortex shedding are very common in high-voltage overhead transmission lines. Although such vibrations are barely perceptible due to their low amplitudes (less than a conductor diameter), they are, however, extremely important since they may lead to conductor fatigue. Mathematical models are therefore necessary for the computation of these vibrations, in order to evaluate the risk of potential damage to the line as well as for studying the efficiency of damping measures. For single conductor lines, the so-called energy-balance method gives good results in estimating the vibration amplitudes. However, the problem becomes more involved for bundled conductors with spacer dampers, commonly used in high-power transmission in many countries, and a modified form of the energy-balance method is presented here. Singular perturbation methods are employed, along with the energetically equivalent standing wave amplitudes obtained from the modified energy-balance methods, to determine the bending strains at critical points. This gives an estimate for the maximum strain levels in a conductor, which can be very useful in the design of transmission lines and for the optimization of the corresponding damping devices.

1. INTRODUCTION

Of the different mechanical vibration phenomena in high-voltage overhead transmission lines, wind-excited oscillations of the conductors constitute a major issue. The most common of these wind-excited vibrations are those generated by vortex shedding in the frequency range of approximately 10–60 Hz. Thus, only these are considered in this paper. Although such vibrations are barely perceptible due to their low amplitudes (less than a conductor diameter), they are, however, extremely important since they may lead to conductor fatigue. Mathematical models are therefore necessary for the computation of these vibrations, in order to evaluate the risk of potential damage to the line as well as for studying the efficiency of damping measures.

The energy-balance method is commonly used to estimate the vibration amplitudes of single conductor lines Hagedorn 1980; Vecchiarelli *et al.*, 1999; a more general overview on the relevant publications can be found in these references. In the present, paper the method is generalized for the case of bundled conductors with spacer dampers. The main ideas of the mathematical model for a single conductor may be briefly described as follows. Consider the schematic representation of a single conductor line shown in Fig. 1. Since the length of the span is typically of the order of $L = 300$–500 m, the wavelengths λ of the vibrations are of the order of a few meters, and the sag only a few percent of the span, the sag can be neglected. Thus, the conductor may be modelled as a taut string with

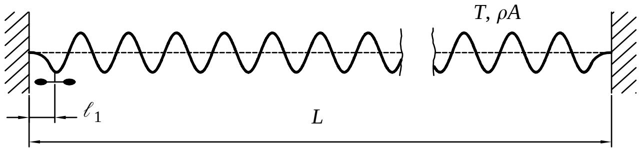

Figure 1. Model of a single conductor with Stockbridge damper.

a very small bending stiffness. It can therefore be described by

$$[EI(x)w'']'' + \rho A\ddot{w} - Tw'' = f_W(x,t) + f_D(w,\dot{w},t), \tag{1}$$

where $w(x,t)$ is the transverse displacement of the conductor at point x and time t (the x-coordinate is measured from the left end of the conductor), EI is the bending stiffness, ρA the mass per unit length, T the tension, f_W designates the aerodynamic forces acting on the conductor, and f_D the forces responsible for structural damping. The maximum vibration amplitudes occur, when the wind blows transversely to the transmission line, with vortex shedding alternating between the upper and the lower edges of the conductor. The conductor will therefore oscillate in a vertical plane, which is the only case considered herein. Moreover, the bending stiffness is small and satisfies the inequality

$$\sqrt{\frac{EI}{T}} \ll \frac{\lambda}{2\pi}. \tag{2}$$

It can therefore be neglected in the dispersion relation, which will be that of an ideal taut string, and is of importance in the bending boundary layers only. These occur at the boundaries and wherever concentrated forces act on the conductor. The term *boundary layer* is used here as defined in singular-perturbation theory. Boundary layers occur normally at boundaries, where the higher-order derivatives in the differential equations become important. Both the boundary layers from fluid mechanics and the bending boundary layers here defined are particular cases. Bending boundary layers are of extreme importance for the calculation of the bending strains, as will be explained in detail in the later sections. The overall vibration levels can however be estimated without paying any attention to the bending boundary layers. The bending strains are then determined afterwards, using these vibration amplitudes, by a simple formula obtained from a perturbation analysis.

Estimates of the vortex-excited vibration levels of transmission lines are traditionally based on worst-case scenarios, due to the temperature dependence of the tension, T, and the variability of the wind velocity field. For these reasons, only vortex shedding due to a wind blowing transversely to the line is considered in more detail.

For the frequency range considered, different types of damping devices are implemented to suppress conductor vibrations; the most common is the Stockbridge damper and related types. The most basic physical model for a Stockbridge damper assumes linear response characteristics and utilizes the damper impedance $Z(s)$, where s is the complex frequency parameter. The complex force amplitude, $\hat{F}$, at the damper clamp can be determined from $\hat{F} = Z\hat{v}$, where $\hat{v}$ is the complex velocity amplitude of the conductor at the clamp. A linear two-degree-of-freedom approximation can be assumed for the damper under certain conditions. An analytical expression thereof has been proposed by Schäfer (1980). The action of the damper on the conductor is illustrated in Fig. 2, where

$$F(t) = T[w'(l_1^+,t) - w'(l_1^-,t)] \tag{3}$$

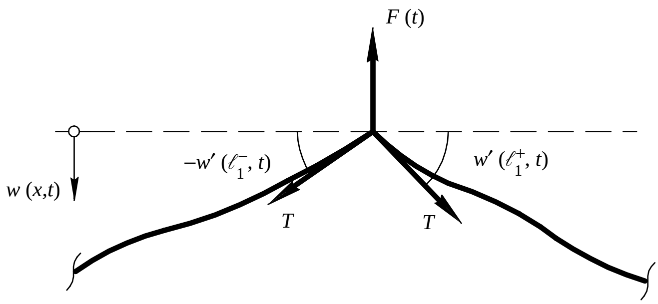

Figure 2. Damper force acting on the conductor.

and in which can be seen that the slope of the conductor displacement is discontinuous at $x = l_1$. More elaborate mathematical models for the damper have also been suggested [e.g., Hagedorn (1982)].

The frequency of both vortex shedding and the corresponding force acting on the conductor in the vertical direction, is proportional to the wind velocity (at least in the laminar regime). Without the Stockbridge damper shown in Figure 1, the eigenfrequencies of a conductor modelled as a taut string with the assumed boundary conditions is $f_n = 0.1\,\mathrm{Hz}$, where $n = 1, 2, 3, \ldots$. This means that the interval ranging approximately between 10 and 60 Hz, in which the vortex-excited oscillations are significant, contains about 500 resonance frequencies. Due to the unknown parameters contained in the problem, it does not make much sense to distinguish between these individual resonances.

2. ENERGY BALANCE FOR SINGLE CONDUCTOR

Rather than using differential equation (1), where the terms f_W (aerodynamic forces) and f_D (structural damping) cannot easily be defined, the maximum possible vibration levels at a given wind velocity (and hence also at a given frequency f) are estimated using the energy-balance equation

$$P_W(A) = P_S(A) + P_D(A), \tag{4}$$

where A is a representative vibration amplitude in the span, P_W the wind power input, P_D the power dissipated in the Stockbridge damper and P_S the power dissipated due to structural damping. The wind power input P_W is computed for a standing harmonic wave of amplitude A over the whole span of length $L - l_1 \simeq L$ using data from wind tunnel experiments (Staubli 1979; Bishop & Hassan 1964; Brika & Laneville 1995; Chen 1987; Diana & Faloc 1971) or experiments carried out with transmission lines in the field by (Rawlins 1983,1998) for laminar and turbulent uniform wind. The structural damping P_S is determined by laboratory experiments (Gutzer 1998; Hagedorn & Kraus 1990), and may be neglected when supplemental damping devices are used on the conductor. The power dissipated in the damper, P_D, is calculated from its impedance and is also a function of conductor parameters and of the distance l_1 (Hagedorn 1980).

Estimating the vibration levels via energy balance (4), therefore, requires the solution of only one nonlinear algebraic equation. The result will be the vibration amplitude, A, as a function of either one of the mutually dependent parameters f (vibration frequency) or v (wind speed), as shown by the solid line in Figure 3. In using an energy balance in this form, the right-hand boundary condition in Figure 1 was not used. Consequently, the discrete spectrum of the eigenvalue problem corresponding to the free vibrations of the

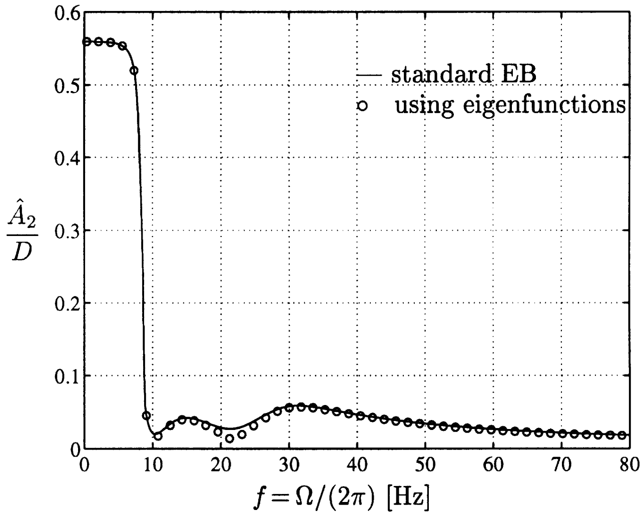

Figure 3. Maximum vibration amplitudes from energy balance (EB): —, standard EB; ∘, using eigenfunctions.

conductor is replaced by a continuous spectrum. Energy balance in the described form has become a standard procedure in vibration studies of single-conductor transmission lines (Doocy *et al.* 1979; Schäfer 1980; Tunstall 1997).

The results obtained from application of energy balance to each mode after solving the pertinent eigenvalue problem for the free damped vibration case of a single conductor with a Stockbridge damper is shown in Figure 3 (Hadulla 2000). The circles correspond to individual (complex) resonant frequencies. It is readily observed that the two forms of the energy balance give very similar results. Of course the vibration obtained amplitudes depend on the values for the wind power input, which in reality is a function of many parameters, such as, e.g., turbulence. Also, different laboratories obtained different values for the wind power input, and this should be kept in mind for practical applications.

3. BUNDLED CONDUCTORS

Bundled conductors are frequently used for transmission lines with voltages above 400 kV. The individual conductors of a bundle are connected by means of spacers, designed in such a way as to maintain a constant separation distance between the conductors, in order to guarantee a constant electrical impedance. The spacers are typically mounted at a relative distance of 40 m. The problem of vortex-excited oscillations in conductor bundles is far more involved than in a single conductor line. In order to avoid vibration islands, rigid spacers may be replaced by spacer dampers, which have certain elastic and damping properties. In addition, Stockbridge dampers may also be used. A schematic view of a spacer damper in a bundle with four conductors (quad bundle) is shown in Figure 4, with the forces acting at one of the conductor clamps shown in Figure 5.

Typically, a spacer damper consists of a rigid central frame to which n rigid arms are connected via viscoelastic elements. In fact, the number of arms can be from 2 to 10, depending upon the electrical voltage. Due to coupling via the spacer dampers, the conductors will not be restricted to the vertical plane. A single spacer damper can be described by its *complex-impedance matrix*, relating the complex velocity amplitudes at the

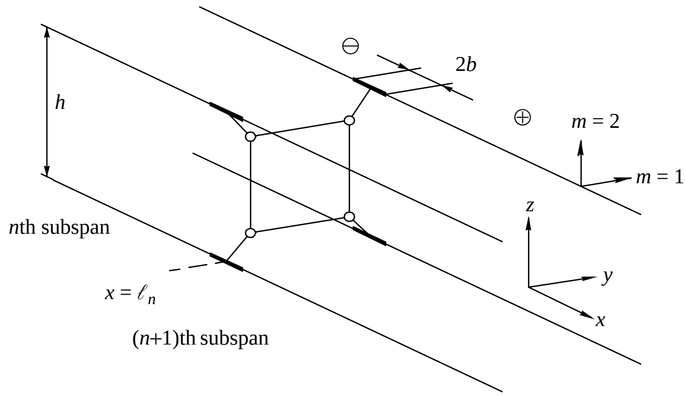

Figure 4. Details of a quad bundle with spacer damper.

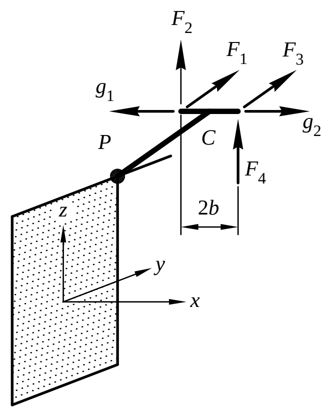

Figure 5. Forces acting at the clamp of a spacer damper.

end points of the clamps to the corresponding forces. For the quad bundle shown in Figure 4, this is a complex 16×16 matrix. The derivation of this impedance matrix as well as criteria for the optimization of spacer dampers are described in (Anderson & Hagedorn, 1995). The dynamics of systems composed of one–dimensional elements has been extensively studied in the literature, see Beloušek (1973) and Skudrzyk (1968) and a more complete list of references therein. In the present case, the bundle vibrations are damped at discrete locations by the spacer dampers, which adds some complications to the problem.

The relative distance between the spacer dampers can no longer be neglected in the energy balance for the bundle. Therefore, as a first step, complex eigenvalues and eigenfunctions are determined. The free vibrations of the conductor between any two spacer dampers are described by the wave equation resulting from equation (1) by neglecting the bending stiffness, as well as aerodynamic forces and structural damping. Separating variables according to

$$w(x, t) = \mathcal{R}e[W(x)e^{st}] \tag{5}$$

leads to

$$W''(x) - \frac{s^2}{c^2}W(x) = 0, \tag{6}$$

with wave velocity $c = \sqrt{T/\rho A}$ and eigenvalue s, so that one has

$$W_{m,n}(x_n) = A_{m,n} \exp\left(\frac{sx_n}{c_m}\right) + B_{m,n} \exp\left(-\frac{sx_n}{c_m}\right), \tag{7}$$

where $A_{m,n}$ and $B_{m,n}$ are complex constants with the index $n = 1, 2, \ldots, N+1$ designating the field between two of the N consecutive spacers, and the index $m = 1, 2, \ldots, 2M$ representative of either any of the two transverse directions for one of the M conductors (in the case of a bundle with M conductors). Since the tension is usually smaller in the lower conductors, the wave speed, c_m, differs from conductor to conductor. Here, new subspan coordinates have been introduced as

$$x_n = x - \ell_{n-1} \quad \text{for} \quad b < x_n < \triangle\,\ell_n - b, \tag{8}$$

where $2b$ refers to the characteristic length of the clamp, which is equal to the clamp width plus the characteristic bending length $\ell_{\text{char}} = \sqrt{EI/T}$. Defining the vector of the velocities at the end points of the clamps of the nth spacer as

$$\dot{\mathbf{w}}_n(t) = \begin{pmatrix} \dot{w}_{1,n}(\triangle\ell_n - b, t) \\ \dot{w}_{2,n}(\triangle\ell_n - b, t) \\ \dot{w}_{1,n+1}(b, t) \\ \dot{w}_{2,n+1}(b, t) \\ \vdots \\ \dot{w}_{2M-1,n+1}(b, t) \\ \dot{w}_{2M,n+1}(b, t) \end{pmatrix}, \tag{9}$$

the relation between the complex force and velocity amplitudes can be formulated using the spacer-impedance matrices as

$$\hat{\mathbf{F}}_n = \mathbf{Z}_n(s)\,\hat{\dot{\mathbf{w}}}_n\,, \quad n = 1, 2, \ldots, N. \tag{10}$$

Together with the boundary conditions at both ends of the span, this finally leads to the eigenvalue problem

$$\mathbf{J}(s)\,\mathbf{a} = \mathbf{0}, \tag{11}$$

where the column matrix $\mathbf{a}$ is given by

$$\mathbf{a} = (A_{1,1}, B_{1,1}, A_{2,1}, B_{2,1}, A_{3,1}, B_{3,1}, \ldots, A_{2M,N+1}, B_{2M,N+1})^{\mathrm{T}}. \tag{12}$$

Note that equation (11) is not a standard matrix eigenvalue problem in the complex frequency parameter s, since the elements of the matrix $\mathbf{J}$ depend explicitly on s. The frequency parameter appears not only in exponential functions but also in the expressions for the impedance matrices, which are contained in $\mathbf{J}$. The spectrum of this eigenvalue problem is very dense and its numerical solution presents considerable numerical problems. In Ramnale (2000), an algorithm is given for the solution of this problem.

With the complex eigenmodes thus determined, the energy balances can now be formulated for each of these eigenmodes. This is done here by equating the total maximum possible wind power input over the whole span to the power dissipated in all the spacer dampers as well as in the conductors. This again leads to a single nonlinear equation, analogous to equation (4), which determines the free amplitude parameter. This simple energy balance only guarantees that the total wind power input is equal to the power dissipated. Other energy-balance equations could also be formulated by considering the wind power input in each field, the energy dissipated in the spacer dampers at its ends, and the energy transport to and from the neighboring fields.

The wind power input is calculated taking into account only the vertical components of the conductor motions. The leeward conductors do experience the vortices shed by the upstream conductors, so that one may expect that the corresponding wind power input differs from one conductor to another. However, this influence is normally disregarded, since the distance between the conductors is usually larger than 10 times their diameter. Most experiments on this phenomenon where carried out in wind tunnels and with oscillating cylinders, and reliable data for vibrating bundles of conductors do not seem to be available. For more detailed references on this important problem see Laneville & Brika (1999). Different wind power inputs for the individual conductors can however be easily incorporated in the calculations, once the data become available.

The power dissipated in the spacer dampers can be calculated using the spacer-impedance matrices, details being given in Mitra (2001), Hadulla (2000) and Hagedorn & Hadulla (2000).

The energy balance for the bundle vibrating according to any one of the complex normal modes then assumes the form

$$\sum_{n=1}^{N+1}\sum_{m=1}^{M} P_{W,n}(\hat{A}_b \rho_{2m,n}) = \sum_{n=1}^{N} P_{D,n}(\hat{A}_b) + \sum_{n=1}^{N+1}\sum_{m=1}^{M} P_{S,n}\left(\hat{A}_b\sqrt{\rho^2_{2m-1,n}+\rho^2_{2m,n}}\right), \tag{13}$$

where $\hat{A}_b$ is the variable to be determined, scaling the vibration amplitudes. The parameters $\rho_{m,n}$ are averaged and normalized vibration amplitudes are determined in each field from the complex coefficients $A_{m,n}$ and $B_{m,n}$ contained in the complex eigenvectors. In the wind power expressions in equation (13), only the vertical component of the vibrations was taken into account. The total vibration amplitude in each subspan and conductor is obtained from

$$\begin{aligned} \hat{A}^{\text{total}}_{m,n} &= \sqrt{\hat{A}^2_{2m-1,n}+\hat{A}^2_{2m,n}} \quad (m = 1, 2, 3, \ldots, M) \\ &= \hat{A}_b\sqrt{\rho^2_{2m-1,n}+\rho^2_{2m,n}}\,. \end{aligned} \tag{14}$$

Figure 6 shows the maximum vibration amplitudes obtained in each field for the case of a span of a bundled conductor with $N = 3$ spacer dampers (with subspan lengths of 60–100–100–60 m) (Mitra 2001). In each field, only the vibration amplitude of the conductor with the highest vibration level is shown at each of the eigenfrequencies. This may occur at a different conductor for each frequency and field.

4. COMPUTATION OF BENDING STRAIN

Up to this point, we have considered that the vibration takes place in a cable with negligible bending stiffness. The formulations that follow consider the effect of flexural rigidity EI at the critical points of the whole span by applying singular perturbation methods at those points. At the point of application of concentrated forces, the behaviour of a string differs considerably from that of a string with finite-bending stiffness. While in the first case, the slope is discontinuous at the point of application of force, the string has a finite curvature if the bending stiffness is taken into account. Therefore, the bending stiffness is taken into account not only at the clamped ends of the suspension clamp but also at the ends of the characteristic length of the clamps, indicated as the boundary

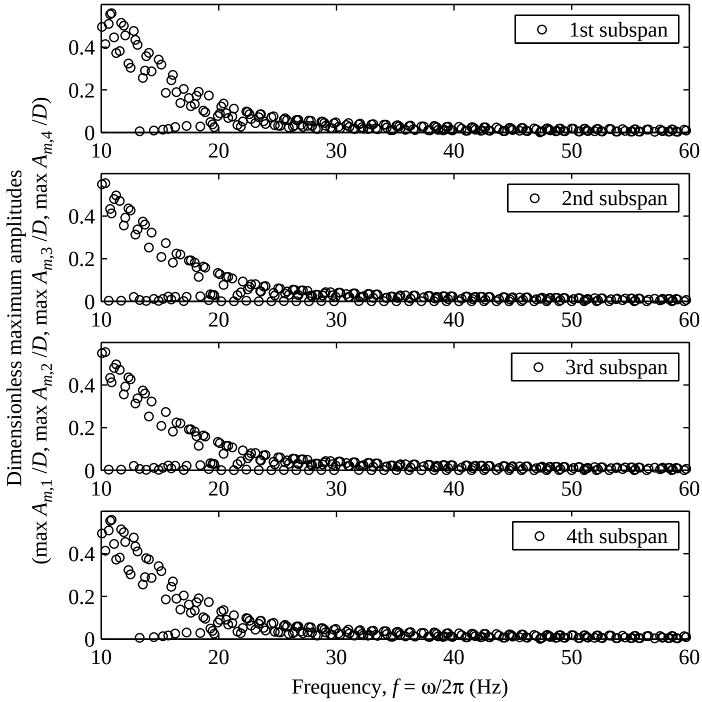

Figure 6. Plot of dimensionless amplitude, expressed as the ratio to the conductor diameter, with respect to frequency.

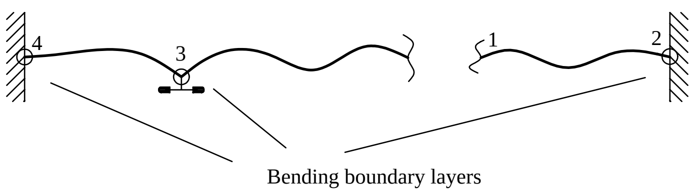

Figure 7. Bending boundary layers of a conductor in a span.

regions in Figure 7. These points are referred to as the critical points. A method which applies singular perturbation techniques in the boundary regions for the case of a single conductor has been explicitly dealt with in Hagedorn (1980). Here, we will apply the resulting equations, along with the vibration amplitudes obtained from the energy-balance method, in order to determine the strains at the critical points.

The differential equation in the boundary regions is

$$[EI(x)w'']'' + \rho A\ddot{w} - Tw'' = 0, \tag{15}$$

where EI is the bending stiffness of the conductor. Here, it should be pointed out that the value of the cross-sectional moment of inertia I to be used in equation (15) is not readily known. Two extreme values for I would be $I_{\min}$, obtained by assuming the individual wires to slide without friction, and $I_{\max}$, obtained by assuming a rigid cross-section with no sliding between the individual wires. According to a recommendation by Scanlan & Swart (1968), a value approaching $EI_{\min}$ should be used, since the wires of the outer layers of the conductor are free to buckle, while the wires of the inner layers tend to act together.

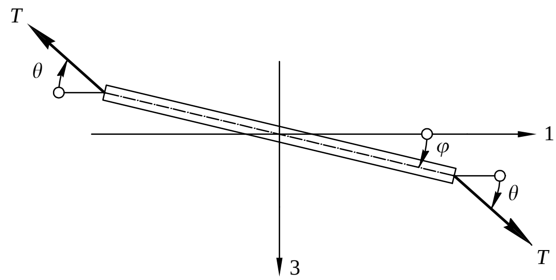

Figure 8. Displacement of the clamp of the spacer damper.

From Hagedorn (1980), the relation

$$w'' = \frac{1}{2}\sqrt{\frac{T}{EI}}(\triangle w'_{\text{string}}) \tag{16}$$

is obtained between the local curvature, w'', of a cable with small bending stiffness and the corresponding discontinuity in slope, $\triangle w'_{\text{string}}$, of a taut string with negligible bending stiffness. This relation follows when singular perturbation theory is applied to a taut cable with small bending stiffness. From laboratory experiments, we know that this first-order approximation gives excellent results for conductors vibrating in higher modes. As shown in Figure 8, the slope that is to be considered for evaluation of the curvature is actually the relative slope between the cable and the clamp.

The curvatures thus obtained are substituted into the equation

$$\varepsilon = \frac{d}{2}w'' \tag{17}$$

to obtain the strain at those critical points. The values of w'' to be used here are obtained from equation (16), using the discontinuity in slope in the corresponding eigenmode, scaled using energy balance. The term d refers to the characteristic diameter, which, due to the complex nature of the cable, will not be equal to the external diameter D of the cable, but will rather lie between D and the diameter of the wires. This characteristic diameter must be determined experimentally. A detailed review of all these steps can be found in Mitra (2001).

In Figure 9, strains at critical points for each of the conductors of a typical four-bundle are plotted against frequency. In this case, we have taken a four-bundle having different cable tensions in the upper and lower cables. The cable tensions in the upper cables are obviously higher than that in the lower ones.

5. CONCLUSION

The energy-balance method has been used not only to estimate the vibration amplitudes of the vortex-excited vibrations of single conductors in overhead transmission lines with dampers, but also to determine the strains at critical points in the span. In modelling the vortex-excited oscillations in this manner, it is assumed that the conductor is always in resonance. The discrete spectrum of the free conductor vibrations is thus replaced by a continuous spectrum. This can be justified by the fact that the resonance frequencies are very closely spaced, and it makes little sense to physically distinguish between the individual resonances, which depend on parameters that are not well determined.

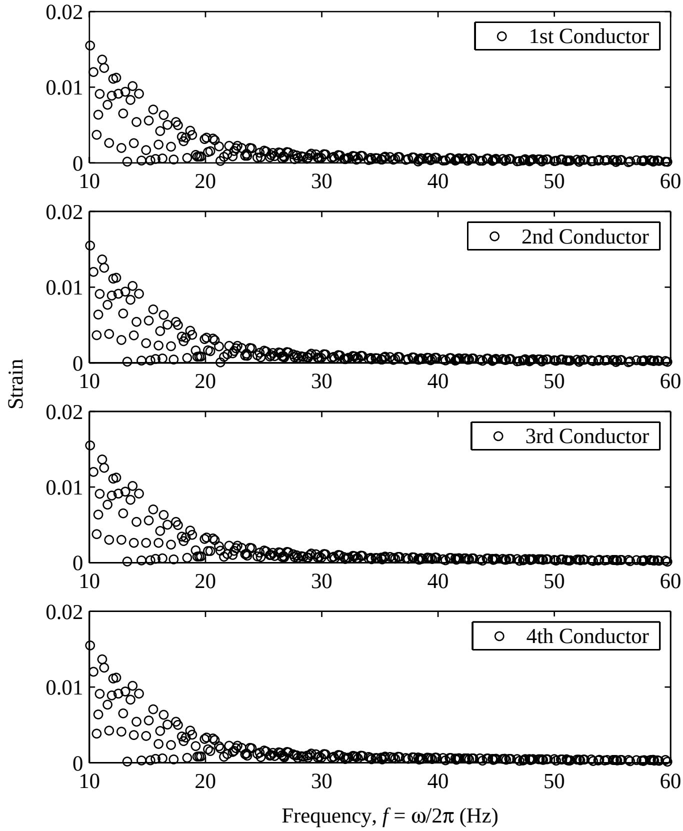

Figure 9. Plot of the maximum strain in a conductor in a bundle with respect to frequency.

In the energy balance, a constant transverse wind velocity is assumed, and the vibration levels are characterized by a single, nominal vibration amplitude describing an ideal standing harmonic wave. The damper action is modelled using its impedance, but more elaborate descriptions of the damper dynamics may also be used. From field tests, it is known that these relatively simple calculations based on energy balance yield realistic and useful results.

At high power levels, the single conductor lines are replaced by bundled conductors for practical reasons. Spacer dampers with elastic and damping properties are used to maintain a certain distance between the individual conductors, in order to guarantee a constant electrical impedance, as well as to damp vibrations. The relative distances between the spacer dampers are important in this case, since they strongly affect the vibration modes, and the discrete spectrum can no longer be represented by a continuous spectrum. In the present paper, it has been demonstrated how the energy-balance method can be adapted to this case. First, the eigenvalue problem is formulated and solved numerically for the bundled conductor and the complex eigenfrequencies and eigenfunctions are obtained, the latter in terms of explicit solutions of the wave equation. The spacer dampers are described by their complex impedance matrices. The resulting eigenvalue problem is of a nonstandard form, since elements of the system matrix contain the complex frequency parameter explicitly in nonalgebraic functions. The energy balance is then carried out for each of the complex eigenmodes. As a result, nominal vibration amplitudes are obtained for each subspan between neighboring spacer dampers and in each conductor for any given wind speed.

Bending strains in the conductors are then computed from these vibration amplitudes in the same way as in single conductor lines. Although the computation times are much larger than in a single conductor line, the modified energy-balance method is still relatively simple to apply. The authors believe that it may be a useful tool for the study of vortex-excited vibrations in conductor bundles and can be used to optimize the damping of this type of vibrations.

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